MAGNETOHYDRODYNAMICS Vol. 50 (2014), No. 3, pp. 263–277

NUMERICAL STUDY OF THE STOKES' FIRST PROBLEM FOR THERMOELECTRIC MICROPOLAR FLUID WITH FRACTIONAL DERIVATIVE HEAT TRANSFER

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In this work, the flow of an electrically conducting thermoelectric micropolar fluid over a suddenly moved heated plate is considered using the methodology of fractional calculus. The governing system of coupled equations in the frame of the boundary layer equations is applied to the unsteady Stokes' first problem. Finite element technique is proposed to analyze the problem. Numerical solutions for temperature, velocity and microrotation distributions are obtained. Numerical results are computed and illustrated graphically. The effects of the flow parameters, such as fractional order , thermoelectric figure-of-merit ZT_0, Seebeck coefficient k_0 and magnetic parameter M on all distributions are studied with the aid of figures. The theories of coupled thermoelectric micropolar fluid and of generalized thermoelectric micropolar fluid with one relaxation time follow as a limit case. Some comparisons have been shown in figures to estimate the effects of various parameters on all studied fields.

Nomenclature.

x, y, z	space coordinates
u	fluid velocity along the x-direction
U	velocity of the plate
N	microrotation component
T	temperature
ρ	density
t	time
Pr	Prandtl number
M	magnetic field parameter
p	pressure
j	microinertia
$\mathbf{q}$	velocity vector
$\mathbf{N}$	microrotation vector
$\mathbf{f}$	body forces per unit mass
$\mathbf{l}$	body couple per unit mass
$\mathbf{H}$	magnetic field intensity vector
$\mathbf{B}$	magnetic induction vector
$\mathbf{E}$	electric field intensity vector

$\mathbf{J}$	conduction electric density vector
k_0	Seebeck coefficient
π_0	Peltier coefficient
H_0	magnetic field component
σ_0	electrical conductivity
μ_0	magnetic permeability
κ_0	thermal conductivity
α	fractional order
λ, μ, ζ	viscosity coefficients
α_0, β, γ	gyro viscosity coefficients

Introduction. Due to the increasing importance of material flow in industrial processing and elsewhere and to the fact that shear behaviour cannot be characterized by Newtonian relationships, a new stage in the evaluation of fluid-dynamic theory is in progress. Eringen [1] proposed a theory of micropolar fluids taking into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation.

The concept of micropolar fluids deals with a class of fluids, which exhibit certain microscopic effects arising from the local structure and micro-motions of the fluid elements. These fluids contain dilute suspensions of rigid macromolecules with individual motions that support stress and body moments and are influenced by spin inertia. The theory of micropolar fluids and its extension to thermomicropolar fluids [2] may form suitable non-Newtonian fluid models, which can be used to analyze the behaviour of exotic lubricants, colloidal suspensions, polymeric fluids, liquid crystals, human and animal blood, and so forth. Ariman *et al.* [3] have written excellent reviews about the applications of micropolar fluids.

Stokes in 1851 and once more Rayleigh in 1911 discussed the fluid motion above the plate independently assuming the fluid to be Newtonian [4]. In the literature, this problem is referred to as the Stokes' first problem. Subsequently in [5] the above problem is considered with Maxwell fluid instead of Newtonian fluid. Many researchers have studied the Stokes' first problem for non-Newtonian fluids with different constitutive equations [6–12].

In all publications quoted above, the heat equation under consideration is taken as a heat equation based on the classical Fourier law so that the heat fluxes propagate at infinite speed. Cattaneo [13] was the first to offer an explicit mathematical correction of the propagation speed defect inherent in the Fourier heat conduction law. Among the few works devoted to applications of the Cattaneo theory to the flow of conducting micropolar fluid in the presence of a transverse magnetic field, we can refer to the works of Ezzat [14–16].

Direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators. Thermoelectric currents in the presence of magnetic fields can cause pumping or stirring of liquid metal coolants in nuclear reactors or stirring of molten metal in industrial metallurgy. The interaction between the thermal and magnetohydrodynamic fields is mutual owing to the alterations in thermal convection and to the Peltier and Thomson effects (although these are usually small). Thermoelectric devices have many attractive features compared with the conventional fluid-based refrigerators and power generation technologies, such as long life, no moving part, no noise, easy maintenance and high reliability. However, their use has been limited by the relatively low performance of existing thermoelectric materials [17].

Liquid metals are considered to be the most promising coolants for high temperature applications like nuclear fusion reactors because of the inherent high thermal diffusivity, thermal conductivity and hence excellent heat transfer characteristics. Lithium is the lightest of all metals and has the highest specific heat per unit mass. Lithium is characterized by large thermal conductivity and thermal diffusivity, low viscosity, low vapor pressure.

Liquid metal in a closed container made of dissimilar metal exposed to a magnetic field is, in general, driven into motion by thermoelectric effects if the interfacial temperature is non-uniform – a situation likely to occur in fusion reactor blankets owing to the high thermoelectric power of lithium. Lithium is the most promising coolant for thermonuclear power installations. Shercliff [18] addresses Hartmann flow and points out the relevance of thermoelectric magnetohydrodynamics (MHD) in liquid metal use, such as lithium, in nuclear reactors.

In the context of magnetizable fluids, Rosensweig [19] shows how the micropolar fluid fits within the framework of thermoelectric ferrohydrodynamics. Recently, Ezzat and Youssef addressed the boundary layer flow of a thermoelectric micropolar fluid over an infinite surface plane under the influence of thermoelectric properties on such a flow [20].

Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, viscoelasticity, biology, physics, and engineering. The most important advantage of using fractional differential equations in these and other applications is their non-local property. It is well known that the integer order differential operator is a local operator, but the fractional order differential operator is non-local. This means that the next state of a system depends not only upon its current state, but also upon all of its historical states. This is more realistic, and it is one reason why fractional calculus has become more and more popular [21–23].

Ezzat [24–26] has established a new model of the fractional heat conduction law by using the new Taylor series expansion of time fractional order which was developed by Jumarie [27]. El-Karamany and Ezzat [28] introduced two general models of the fractional heat conduction law for an inhomogeneous anisotropic elastic solid. Uniqueness and reciprocal theorems are proved and a convolutional variational principle is established and used to prove a uniqueness theorem with no restriction on the elasticity or thermal conductivity tensors except symmetry conditions. For fractional thermoelasticity, El-Karamany and Ezzat [29] established uniqueness, reciprocal theorems and convolution variational principle. The factional order theory of a perfect conducting thermoelastic medium was investigated by Ezzat and El-Karamany [30]. Youssef [31] introduced another new model of fractional heat conduction equation, proved the uniqueness theorem and presented one-dimensional application in the context of thermoelasticity.

The exact solution of the governing equations of the thermofluid theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. To calculate the solution of general problems, a numerical solution technique is used. For this reason, the finite element method is chosen [32].

In the current work, the Stokes' first problem of an electrically conducting micropolar flow over a suddenly moved heated plate in the presence of a transverse magnetic field normal to the plate is considered in the context of a new consideration of heat conduction with fractional order. Finite element methods are applied. Numerical results are given and illustrated graphically for the problem considered. Some comparisons are shown in figures to estimate the effects of the fractional order parameter and thermoelectric properties on the fluid flow.

The results obtained herein are compared with the results predicted by different theories.

1. Problem formulation. Let us suddenly impart a velocity $U(t)$ to the plate in its own plane in the presence of a magnetic field $\mathbf{B}$ and applied on the plane surface. The basic equations in vector form for an incompressible electrically conducting micropolar fluid with the thermal relaxation τ_0 are [20, 22]:

the continuity equation

$$\operatorname{div}(\mathbf{V}) = 0; \tag{1}$$

the momentum equation

$$\rho\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \rho\mathbf{f} - \operatorname{grad}(p) + \zeta\operatorname{rot}(\mathbf{N}) - (\mu+\zeta)\operatorname{rot}\operatorname{rot}(\mathbf{V}) + (\lambda + 2\mu + 2\zeta)\operatorname{grad}\operatorname{div}(\mathbf{V}) + (\mathbf{J})\wedge(\mathbf{B}); \tag{2}$$

the angular momentum equation

$$\rho j\frac{\mathrm{d}\mathbf{N}}{\mathrm{d}t} = \rho\mathbf{l} - 2\zeta\mathbf{N} + k\operatorname{rot}(\mathbf{V}) - \gamma\operatorname{rotrot}(\mathbf{N}) + (\alpha_0+\beta+\gamma)\operatorname{grad}\operatorname{div}(\mathbf{N}); \tag{3}$$

the generalized heat equation with the fractional derivative

$$\rho C_p\left[\frac{\partial}{\partial t} + (\mathbf{N}\cdot\nabla)\right]\left[1+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right]T = \kappa\nabla^2 T - \nabla\cdot\Pi\mathbf{J}, \tag{4}$$

where the Caputo fractional derivative is given by the relation

$$D_t^\alpha f(\bar{x},t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\xi)^{-\alpha}\frac{\partial f(\bar{x},\xi)}{\partial\xi}\mathrm{d}\xi, \quad \alpha\in[0,1];$$

the generalized Ohm's law

$$\mathbf{J} = \sigma_0\left[\mathbf{E} + \mathbf{V}\times\mathbf{B} - S\operatorname{grad}T\right]. \tag{5}$$

The material constants λ, μ, ξ are the viscosity coefficients, and α, β, γ are the gyro viscosity coefficients. These constants confirm to the inequalities

$$\zeta\ge 0,\quad 2\mu+\zeta\ge 0,\quad 3\lambda+2\mu+\zeta\ge 0,$$

$$\gamma\ge 0,\quad |\beta|\le\gamma,\quad 3\alpha_0+\beta+\gamma\ge 0.$$

Let us consider the laminar flow of an incompressible conducting micropolar fluid above a conducting half-space $y>0$. Let the positive y-axis of the Cartesian coordinate system run in the upward direction, the fluid flow through the half-space $y>0$ above and in contact with a flat plate occupying the xz-plane. A constant magnetic field of strength H_0 acts in the z-direction. The induced electric current due to the motion of the fluid that is caused by the buoyancy forces does not distort the applied magnetic field. The previous assumption is reasonably true if the magnetic Reynolds number of the flow ($\mathrm{Rm} = UL\sigma_0\mu_e$) is assumed to be small, which is the case in many aerodynamic applications, where rather flow velocities and electrical conductivities are involved. Under these conditions, no flow occurs in the y- and z-directions and all the considered functions at the given point in the half-space depend only on its y-coordinate and time t. The velocity field is of the form $V\equiv(u,0,0)$ and the microrotation field will be in the form $N\equiv(0,0,N)$. The flow is assumed to be driven by the motion of the flat plate and not by any